

Quantum scaling laws in the onset of dynamical delocalization

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We study the destruction of dynamical localization, experimentally observed in an atomic realization of the kicked rotor, by a deterministic Hamiltonian perturbation, with a temporal periodicity incommensurate with the principal driving. We show that the destruction is gradual, with well defined scaling laws for the various classical and quantum parameters, in sharp contrast with predictions based on the analogy with Anderson localization.

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Quantum chaos is defined as the dynamical behavior of a *quantum* system whose *classical* limit is chaotic. This has triggered a large number of studies trying to relate classical properties to quantum properties, e.g. Lyapunov exponents to quantum fidelity [1, 2], or to detect quantum stability in a quantum-chaotic system [3].

Quantum-chaotic dynamics manifest themselves by characteristic behaviors in which quantum interference plays an important role, making the dynamics distinct from classical dynamics. An example, that shall concern us particularly here, is *dynamical localization* (DL) [4, 5], observed in time-periodic systems. DL is the suppression of the classical chaotic diffusion by quantum interference due to long-range coherence in momentum space; it manifests itself after a typical “localization time” as an exponential localization of the average momentum distribution. Because the system is time-periodic, one can use the Floquet theorem to build a basis of quasi-eigenstates (states that are left unchanged, except for a phase factor, under the temporal evolution over one period). This makes it possible to map the quasi-eigenstates of the time-periodic system on the true eigenstates of a quasi-random static one-dimensional system, which presents the non-trivial Anderson localization. Anderson (or strong) localization has been a major subject in physics in the last decades, with implications in several areas, beyond the primary field of solid state physics [6, 7, 8, 9]. In this paper, we show that studying the breakdown of dynamical localization may also bring some new insight on the physics of Anderson localization. The latter is known to be strongly dependent on the number of freedoms, with marginal localization in dimension 2 and the coexistence of localized and delocalized states – depending on the parameters – in dimension 3. By playing with the temporal dependence of the Hamiltonian – for example by adding incommensurate frequencies to make a quasi-periodic Hamiltonian – it is possible to study temporal equivalents of the Anderson model in various dimensions. What happens if we introduce progressively

a second (incommensurate) frequency in the system, by increasing its strength from zero? As the system is quasi-periodic with two basic frequencies, it is reasonable to assume that it can be mapped onto a two-dimensional Anderson model [10, 11], which, for a small perturbation, is a quasi-1D model, and one could then expect localization to be preserved, at the cost of an increased localization length. Theoretical studies based on the analogy with Anderson localization, supplemented by numerical simulations, indeed predict that the onset of dynamical delocalization takes place when a quasi-periodic perturbation with finite *non-zero* strength is applied on the system [12]. In the present work, we show experimentally that this is *NOT* the case, and that DL is destroyed as soon as the perturbation is non zero, and unravel the scaling laws which govern the phenomenon.

We consider an atomic version of the kicked rotor, a paradigmatic system for theoretical and experimental studies of classical [13] and quantum chaos [14, 15, 16, 17], which consists in exposing laser-cooled atoms to short, periodic pulses of a far-detuned standing wave, so as to obtain an atomic equivalent of the kicked rotor. Using this system, DL has been unambiguously observed and its characteristics studied [18]. The temporal periodicity is a key ingredient. For example, random fluctuations on the strengths of the successive kicks, have been experimentally shown to destroy DL [14], even for fluctuations not significantly affecting the classical diffusive behaviour. Similarly, the introduction of a small amount of non-Hamiltonian evolution – spontaneous emission and the associated random recoil of the atom in the experiment [14, 15] – is enough to induce decoherence, and thus to reduce or kill quantum interference effects, thus restoring the classical dynamics.

In previous works, we have extended the study of the kicked rotor to the two-frequency quasiperiodic case by adding a second series of kicks: the laser-cooled atoms interact with a modulated standing wave of wavevector $\mathbf{k}_L = k_L \mathbf{x}$ forming two series of kicks at frequencies $f_1 =$

$1/T_1$ (primary series) and $f_2 = rf_1$ (secondary series), so as to obtain a system described by the Hamiltonian:

$$H = \frac{P^2}{2} + \sin \theta \left[K \sum_{n=0}^{N-1} \delta(t-n) + aK \sum_{n=0}^{rN-1} \delta\left(t - \frac{n}{r}\right) \right] \quad (1)$$

where we measure momentum in units of $2\hbar k_L$, $\theta = 2k_L x$, time in units of T_1 . The normalized kick amplitude is $K = \Omega_1^2 \hbar k_L^2 \tau T_1 / (2M\Delta)$ (Ω_1 is the resonant Rabi frequency, proportional to the light intensity and τ is the duration of the kicks [23]). In such units, the normalized Planck constant, describing the “quanticity” of the system, is $\hbar = 4\hbar k_L^2 T_1 / M$; it can thus be controlled by changing the frequency of the kicks. We have shown that, in the quasiperiodic case (r irrational), with $a = 1$, DL is destroyed [17].

What are the scaling laws for the onset of delocalization? In order to understand the origin of such laws, we analyze perturbatively the effect of the second series. The effect of each individual kick is expressed by a unitary evolution operator:

$$U(a, K, \hbar) = \exp\left(-i \frac{aK \sin \theta}{\hbar}\right). \quad (2)$$

For sufficiently small a – such that $aK/\hbar \ll 1$ – this operator is close to unity and a single kick only slightly modifies the atomic state. It is the accumulation of a series of small kicks which significantly perturbs the dynamics. If the ratio r of the two frequencies is sufficiently far from any simple rational number, the second series of kicks is applied at quasi-random phases (measured with respect to the principal sequence), so that there is no coherent action of consecutive kicks. In classical language, the positions θ at consecutive secondary kicks are uncorrelated. In the unperturbed Floquet basis, the *incoherent* cumulative effect of the secondary kicks results in a diffusive process. The strength of a single kick being proportional to aK/\hbar , the incoherent cumulative effect of n secondary kicks is proportional to $na^2 K^2 / \hbar^2$, and the characteristic time scale for the effect of the secondary kick series then scales as $T_2 \hbar^2 / (a^2 K^2)$. The other important time scale in the problem is the localization time, scaling like $T_1 K^2 / \hbar^2$. If over one localization time, the effect of the second kick sequence is small, DL has time to establish before being destroyed. In the opposite situation, diffusion in the Floquet basis is the dominant process and no localization is expected. The cross-over between the two regimes arrives when the two time scales are comparable, i.e. when $T_2 \hbar^2 / (a^2 K^2) \simeq T_1 K^2 / \hbar^2$, or (assuming r is of the order of unity):

$$\tilde{a} = \frac{aK^2}{\hbar^2} \simeq 1 \quad (3)$$

\tilde{a} thus represents the scaled parameter governing the onset of delocalization. It depends on both the “chaoticity”

parameter K and the “quanticity” parameter, the effective Planck constant \hbar , which shows the intrinsic quantum nature of the phenomenon. Note that the preceding discussion establishes the expression for the relevant parameter \tilde{a} , but is not sufficient for knowing whether there is an abrupt (as predicted from the Anderson model) or a smooth transition (as we experimentally observe here) between localization and delocalization.

The experimental setup has been described in detail elsewhere [19, 20, 21]. Cesium atoms are first trapped and cooled in a standard Magneto-Optical trap, down to a temperature around $3 \mu\text{K}$. The trap is turned off, and the atoms interact with the doubly-pulsed standing wave [Eq. (1)]. Raman stimulated transitions are then used to measure the population $\Pi(P)$ of a given momentum class, which can be chosen by changing the Raman detuning. It is very easy to directly measure the degree of localization of the system by measuring the population in the zero momentum class $\Pi_0 = \Pi(P = 0)$. As the number of atoms in a given experiment is constant, this value is smaller if the momentum distribution is larger, that is $\Pi_0 \propto \langle \Delta P^2 \rangle^{-1/2}$. In practice, in order to improve the signal to noise ratio, we measure the fraction of atoms with velocity in a small range around zero. The range is comparable to the width of the initial velocity distribution (few recoil velocities) and much smaller than the final width. In our experiment, the standing wave is obtained from a SDL MOPA (Master Oscillator Power Amplifier) delivering around 350 mW. The beam is transported through polarization-maintaining optical fibers to the interaction region. A diode laser mounted in extended cavity configuration and locked on an invar Fabry-Perot interferometer serves as master. The frequency is continuously monitored by an Advantest Q8326 lambda-meter.

In order to study the destruction of DL, we choose a number of kicks that is larger than the localization time $N_L \propto (K/\hbar)^2$ and measure Π_0 for increasing values of a from zero to 0.25. Fig. 1 displays the typical results for seven sets of parameters, that are shown in table I.

Numerical simulations of the kicked rotor quantum dynamics are useful for a detailed interpretation of the results. A few complications must be included in our simulations, which are, ordered by decreasing importance: the finite temporal length of the pulses which makes the kicks slightly different from δ -kicks, the spatial variation of the laser intensity across the atomic cloud which implies that all the atoms do not feel the same K value and some residual spontaneous emission. Altogether, they affect the shape of the curves in a rather limited way: the decay of Π_0 with a is slower by about 20%. Fig. 1 shows a comparison of the numerical calculation for the four curves at $K = 6.8$ and various \hbar values with the corresponding experimental curves: the agreement is very good. There is no adjustable parameter, all the quantities have

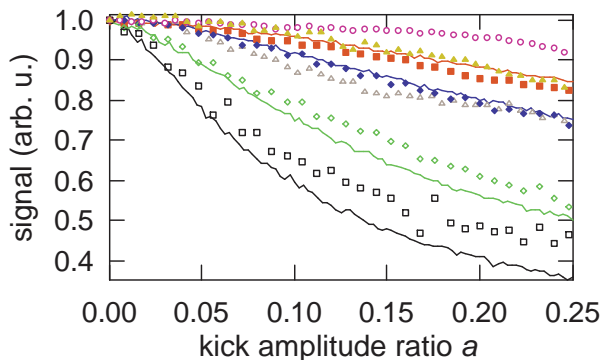


FIG. 1: (Color online) Normalized number of zero-velocity atoms as a function of the amplitude ratio a of the secondary sequence of kicks to the principal sequence of kicks for various values of \tilde{k} and K . In order to ease the comparison, the curves have been normalized so that the value at $a = 0$ is 1 for all curves. The parameters and plotting conventions are listed in table I. The solid lines are numerical simulations for the four curves at $K = 6.8$, with no adjustable parameter, as all quantities of interest have been measured experimentally. The kick strength at the center of the atomic cloud is calculated from the laser power and the geometry of the beam, the temporal profile of the kicks is measured and spontaneous emission rates are calculated from the laser intensity, the detuning and the oscillator strength of the atomic transition.

been either directly measured or calculated from measured quantities and first principles. Small deviations are observed for the lowest curve. This is probably an experimental artefact due to the long duration of the full kick sequence (2.8 ms, compared to 1.2 ms for the upper curve). The atoms are freely falling due to gravity, and, as they escape the central region of the Gaussian-profiled standing wave, they see a smaller light intensity and the momentum diffusion is stopped, and thus the delocalization. For the lowest curve, the motion of the atoms is almost 6 times larger than for the highest curve.

In the various series, a constant ratio $N/N_L = 2.5$ (N_L is the localization time) is kept in order to insure that all experiments correspond to the same “localization stage”. The various sets of parameters have been chosen to allow us to vary either \tilde{k} or K keeping all other parameters constant. We choose $r = 0.681$, a typical “irrational” number, i.e. far from any simple rational, in order to avoid DL and sub-Fourier resonances [22] which are observed in a narrow range around rational numbers.

Data in Fig. 1 clearly demonstrate that the destruction of DL by a second series of kicks is gradual and certainly not a phase transition. The various curves display a qualitatively similar behaviour, a signature of universality in the destruction of DL. In order to exhibit this universal behaviour, we show in Fig. 2 the same data plotted as a function of the scaled amplitude \tilde{a} , given by Eq. (3). The seven experimental curves now coincide, which proves that \tilde{a} is the truly relevant parameter.

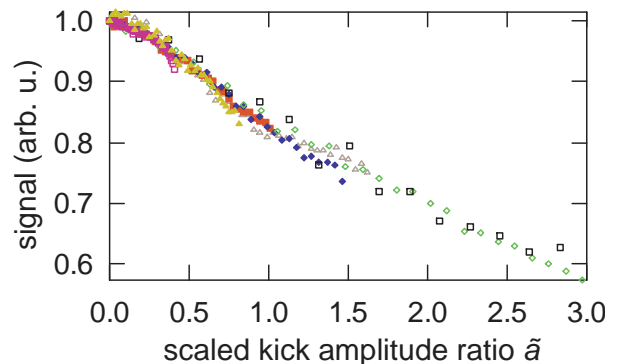


FIG. 2: (Color online) Number of zero-velocity atoms as a function of the scaled kick series amplitude ratio, $\tilde{a} = aK^2/\tilde{k}^2$. One observes the superposition of all curves. Parameters and graphic conventions are the same as in Fig. 1.

f_1 (kHz)	Δ (GHz)	P (mW)	N	N_L	K	\tilde{k}	Symbol
30.000	-18.8	95	35	14	6.8	3.46	■
36.000	-15.6	95	50	20	6.8	2.88	◆
54.000	-10.5	95	113	45	6.8	1.92	◇
72.000	-7.9	95	200	79	6.8	1.44	□
30.000	-21.3	62	18	7	4.5	3.46	○
30.000	-21.3	87	35	14	6.3	3.46	▲
30.000	-21.3	123	70	28	8.9	3.46	△

TABLE I: Sets of parameters used in the curves of Fig. 1. The parameters $r = f_2/f_1 = 0.681$ and the ratio $N/N_L \approx 2.5$ (N_L is the localization time) are the same for all data series. The pulse duration τ is $0.6 \mu\text{s}$ for the 4 top lines and $0.7 \mu\text{s}$ for the bottom 3 ones.

Although the preceeding results are clear-cut proofs that the second series of kicks gradually reduce the localization of the system, this may result from two completely different mechanisms: either the second series destroys the DL and restores a diffusive behavior of the quantum system, with a diffusion constant smoothly increasing from zero (for vanishing a), or the localization is preserved, but with a localization length smoothly increasing with a . Which of two scenarios, the diffusive scenario or the Anderson scenario, correctly describes the physics at work, cannot be decided from the preceding results. This issue can be solved by looking at the momentum distribution. Indeed, the Anderson localized regime is characterized by an exponential localization of the wavefunction in momentum space, while the diffusive regime is associated with a Gaussian momentum distribution. Fig. 3 shows the experimentally observed momentum distribution for various values of \tilde{k} and $K = 6.8$. DL is clearly observed for the smaller value of the scaled amplitude $\tilde{a} = 0.97$ ($\tilde{k} = 3.46$, exponential shape) whereas the larger value $\tilde{a} = 5.6$ is clearly in the diffusive regime ($\tilde{k} = 1.44$, Gaussian shape). The two other plots present intermediate shapes between a Gaussian and an exponential. We thus conclude that the diffusive scenario is the correct one.

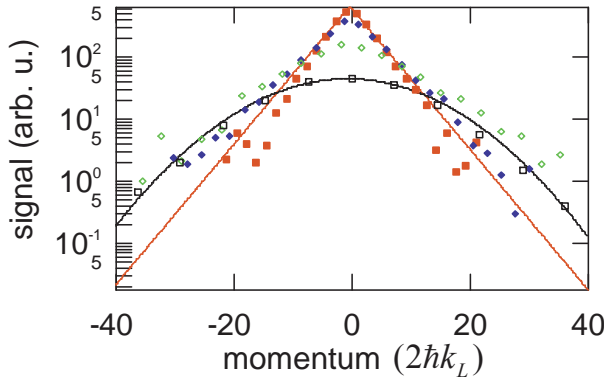


FIG. 3: (Color online) Experimentally observed momentum distributions at $a = 0.25$ in log scale, for various values of k and constant $K = 6.8$. The curve for $k = 3.46$ which corresponds to $\tilde{a} = 0.97$ is well fitted by an exponential, whereas the curve with $k = 1.44$, or $\tilde{a} = 5.6$ is fitted by a Gaussian.

This is somehow surprising, as theoretical arguments and numerical calculations [10, 11, 12] on a slightly different quasi-periodic system – where a single series of kicks has a periodically modulated (at a incommensurate frequency) amplitude – show that the Anderson scenario applies. A possible explanation of this apparent incompatibility might be that quasi-periodicity with two incommensurate frequencies in the driving of the system is not sufficient to determine whether the system is localized or not. In other words, quasi-periodic driving of a Hamiltonian system might not lead to universal behaviour. This is a rather difficult theoretical problem, never treated in the literature, to the best of our knowledge. Experiments on the quasi-periodically driven atomic rotor may help to clarify this stimulating issue.

In conclusion, we have observed that the destruction of dynamical localization by the addition of a small Hamiltonian periodic perturbation at a frequency incommensurate with the principal driving, leads to a gradual destruction of the localization, through a continuous growth of a residual diffusion constant, and *NOT* to the equivalent of Anderson localization in a two degrees of freedom system. We have also determined and tested the quantum scaling laws governing the onset of delocalization.

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- [1] R. A. Jalabert and H. M. Pastawski, Phys. Rev. Lett. **86**, 2490 (2001).
- [2] F. M. Cucchiatti, D. A. R. Dalvit, J. P. Paz, and W. H. Zurek, Phys. Rev. Lett. **91**, 210403 (2003).
- [3] S. Schlunk, M. B. d'Arcy, S. A. Gardiner, D. Cassettari, R. M. Godun, and G. S. Summy, Phys. Rev. Lett. **90**, 054101 (2003).
- [4] G. Casati, B. V. Chirikov, F. M. Izrailev, and J. Ford, *Stochastic behavior of a quantum pendulum under a periodic perturbation* (Springer-Verlag, Berlin, Germany, 1979), vol. 93 of *Lecture Notes in Physics*, p. 334.
- [5] F. M. Izrailev, Phys. Rep. **196**, 299 (1990).
- [6] P. W. Anderson, Rev. Mod. Phys. **50**, 191 (1978).
- [7] D. R. Grempel, R. E. Prange, and S. Fishman, Phys. Rev. A **29**, 1639 (1984).
- [8] E. Abrahams, P. W. Anderson, D. C. Licciardello, and T. V. Ramakrishnan, Phys. Rev. Lett. **42**, 673 (1979).
- [9] B. Kramer and A. Mackinnon, Rep. Prog. Phys. **56**, 1469 (1993).
- [10] D. L. Shepelyansky, Physica D **8**, 208 (1983).
- [11] E. Doron and S. Fishman, Phys. Rev. **60**, 867 (1988).
- [12] G. Casati, I. Guarneri, and D. L. Shepelyansky, Phys. Rev. Lett. **62**, 345 (1989).
- [13] B. V. Chirikov, Phys. Rep. **52**, 263 (1979).
- [14] B. G. Klappauf, W. H. Oskay, D. A. Steck, and M. G. Raizen, Phys. Rev. Lett. **81**, 1203 (1998).
- [15] H. Ammann, R. Gray, I. Shvachuck, and N. Christensen, Phys. Rev. Lett. **80**, 4111 (1998).
- [16] M. K. Oberthaler, R. M. Godun, M. B. d'Arcy, G. S. Summy, and K. Burnett, Phys. Rev. Lett. **83**, 4447 (1999).
- [17] J. Ringot, P. Szriftgiser, J. C. Garreau, and D. Delande, Phys. Rev. Lett. **85**, 2741 (2000).
- [18] F. L. Moore, J. C. Robinson, C. Bharucha, P. E. Williams, and M. G. Raizen, Phys. Rev. Lett. **73**, 2974 (1994).
- [19] P. Szriftgiser, H. Lignier, J. Ringot, J. C. Garreau, and D. Delande, Commun. Nonlin. Sci. Num. Simul. **8**, 301 (2003).
- [20] J. Ringot, P. Szriftgiser, and J. C. Garreau, Phys. Rev. A **65**, 013403 (2002).
- [21] J. Ringot, Y. Lecoq, J. C. Garreau, and P. Szriftgiser, Eur. Phys. J. D **7**, 285 (1999).
- [22] P. Szriftgiser, J. Ringot, D. Delande, and J. C. Garreau, Phys. Rev. Lett. **89**, 224101 (2002).
- [23] In the real experiment the standing wave pulses have a finite duration of $\tau = 600$ ns (of square shape). They can be considered as δ functions as far as the motion of the atom during the pulse is negligible compared to the spatial period of the potential, that is for $\langle p \rangle \tau / M \ll \lambda_L / 2$ (p is the momentum and M is the mass of the atom), condition that is easily satisfied in the experiment.

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